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Robust H_∞ Fault–tolerant Control against Sensor and Actuator Failures for Uncertain Descriptor Systems

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Abstract

Based on H_∞ theory in descriptor systems, a method of designing output-feedback dynamical compensators for uncertain descriptor systems with sensor and actuator failures is presented. The dynamical compensator designed by the proposed method can guarantee that the resultant closed-loop uncertain descriptor system is regular, impulse-free, stable and keeps certain H_∞ norm performance in the normal condition as well as in the event of sensor and actuator failures. A numerical example shows the effect of the proposed method.

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1. Introduction

Descriptor systems capture the dynamic behaviour of many natural phenomena, and have wide applications in many fields, such as network theory and robotics, and have attracted much attention in recent years. In practical applications, parameter uncertainties contained in the dynamical behaviour of many physical processes are unavoidable and failures of control components often occur. Therefore to design robust fault-tolerant controllers for uncertain descriptor systems is necessary. There are some

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articles to discuss the problem of state-feedback robust fault-tolerant control for descriptor systems, see for instance [1-4]. For state-feedback control approaches, all system state variables should be available. However, this is not the case for most practical systems. For certain design problems, the usual static feedback controllers cannot meet the required design criteria. Therefore dynamical compensators are considered. In the other hand, these papers only consider sensor failures or actuator failures. However few articles so far appear to discuss fault-tolerant control against simultaneous failures of sensor and actuator by dynamical compensators for uncertain descriptor systems.

2. Problem Formulation

Consider the following descriptor linear system

$$\begin{cases} E\dot{x} = [A + \Delta A(\sigma)]x + Bu + Gw_0 \\ y = Cx + w_1 \\ z = \left[(Hx)^T \quad u^T\right]^T \end{cases}, \tag{1}$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$, $y \in \mathbf{R}^p$ and $z \in \mathbf{R}^q$ are the state vector, the input vector, the measured output vector, and the controlled output vector, respectively; $w_0 \in \mathbf{R}^s$ and $w_1 \in \mathbf{R}^p$ are the disturbance input vectors; $E, A \in \mathbf{R}^{n\times n}$, B, C, H and G are known matrices of appropriate dimensions with $\mathrm{rank}E = r \le n$; $\Delta A \in \mathbf{R}^{n\times n}$ is a uncertain parameter matrix and is assumed to be of the following form:

$$\Delta A = ML(\sigma)N, \tag{2a}$$

where σ is an uncertain parameter vector, $L(\sigma)$ satisfies

$$L(\sigma)^T L(\sigma) \le \rho^2 I, \tag{2b}$$

and M , N are known real constant matrices with appropriate dimensions. For convenience, ΔA is said to be admissible uncertainty if (2) holds.

We consider an output-feedback dynamical compensator in the following form:

$$\begin{cases} E\dot{\xi} = \hat{A}\xi + \hat{B}y \\ u = K\xi \end{cases}. \tag{3}$$

Applying (3) to system (1), gives the following closed-loop descriptor system:

$$\begin{cases} E\dot{x} = (A + \Delta A)x + BK\xi + Gw_0 \\ E\dot{\xi} = \hat{A}\xi + \hat{B}Cx + \hat{B}w_1 \\ z = \left[(Hx)^T \quad (K\xi)^T\right]^T \end{cases}. \tag{4}$$

Let

$$e = x - \xi, \; M_0 = \begin{bmatrix} M \\ M \end{bmatrix}, \; N_0 = \begin{bmatrix} N & 0 \end{bmatrix}.$$

Then system (4) can be rewritten as

$$\begin{cases} E_c\dot{x}_c = (A_c + \Delta A_c)x_c + G_c w_c \\ z = H_c x_c \end{cases}, \tag{5}$$

where

$$x_c = \begin{bmatrix} x \\ e \end{bmatrix}, \; w_c = \begin{bmatrix} w_0 \\ w \end{bmatrix}, \; E_c = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \; G_c = \begin{bmatrix} G & 0 \\ G & -\hat{B} \end{bmatrix}, \; H_c = \begin{bmatrix} H & 0 \\ K & -K \end{bmatrix},$$

$$A_c = \begin{bmatrix} A+BK & -BK \\ A+BK-\hat{A}-\hat{B}C & \hat{A}-BK \end{bmatrix}, \ \Delta A_c = M_0 L N_0 .$$

Now, concepts of sensor and actuator failures are discussed. The system (1) has p sensors and m actuators, denoted by $\Omega = \{1,2\cdots,p\}$ and $\Sigma = \{1,2\cdots,m\}$, respectively. Define a set of sensor failures as $\Lambda \subset \Sigma$ of system (1) , and a set of actuator failures as $F \subset \Omega$, Introduce the following decomposition

$$B = B_F + B_{\overline{F}}, \ C = C_\Lambda + C_{\overline{\Lambda}}, \tag{6}$$

where B_F is formed from B by zeroing out columns corresponding to F, C_Λ formed from C by zeroing out rows corresponding to Λ, $B_{\overline{F}} = B - B_F$ and $C_{\overline{\Lambda}} = C - C_\Lambda$. Similar to (6), we have the following decomposition

$$K = K_F + K_{\overline{F}}, \ \hat{B} = \hat{B}_\Lambda + \hat{B}_{\overline{\Lambda}}, \tag{7}$$

where K_F is formed from K by zeroing out rows corresponding to F, and $\hat{B}_\Lambda$ formed from $\hat{B}$ by zeroing out columns corresponding to Λ.

In the case of sensor and actuator failures according to Γ and F respectively, system (1) and (5) may be rewritten as

$$\begin{cases} E\dot{x} = Ax + B_F u + Gw_0 \\ y = C_\Lambda x + w \\ z = \begin{bmatrix} (Hx)^T & u^T \end{bmatrix}^T \end{cases} \tag{8}$$

and

$$\begin{cases} E_c \dot{x}_c = (A_{cF} + \Delta A_c) x_c + G_{cF} w_c \\ z = H_{cF} x_c \end{cases}, \tag{9}$$

where

$$A_{cF} = \begin{bmatrix} A+B_F K_F & -B_F K_F \\ A+B_F K_F-\hat{A}-\hat{B}_\Lambda C_\Lambda & \hat{A}-B_F K_F \end{bmatrix}, \ G_{cF} = \begin{bmatrix} G & 0 \\ G & -\hat{B}_\Lambda \end{bmatrix}, \ H_{cF} = \begin{bmatrix} H & 0 \\ K_F & -K_F \end{bmatrix}.$$

For convenience, we call system (1) the normal descriptor linear system and system (5) the normal closed-loop descriptor system. Correspondingly, (8) is called the fault descriptor system and (9) is called the fault closed-loop descriptor system.

With the above description, the problem about H_∞ fault-tolerant control for uncertain descriptor systems to be studied in this paper can be described as follows.

Problem RFT (Robust Fault-Tolerant)**:** Given the descriptor system (1), the system (8) and a constant $a > 0$, determine real matrices $K \in \mathbf{R}^{m\times n}$, $\hat{A} \in \mathbf{R}^{n\times n}$ and $\hat{B} \in \mathbf{R}^{n\times p}$ in the output-feedback dynamical compensator (3) such that the normal closed-loop system (5) and the fault closed-loop system (9) are stable, impulse-free, regular for all admissible uncertainties and the transfer functions

$$T_c = H_c(sE_c - A_c - \Delta A_c)G_c$$

and

$$T_{cF} = H_c(sE_c - A_{cF} - \Delta A_c)G_{cF}$$

satisfy $\|T_c\| < a$ and $\|T_{cF}\| < a$, respectively.

3. Robust H_∞ Fault-Tolerant Controller Design

First, let us introduce some concepts that are fundamental in our development.

Definition 1：(i) The descriptor system (1) or the matrix pair (E, A) is regular if $\det(sE - A)$ is not identically zero.(ii) The descriptor system (1) or the matrix pair (E, A) is impulse-free if $\deg \det(sE - A) = \mathrm{rank}E$. (iii)The descriptor system (1) or the matrix pair (E, A) is admissible, if it is regular, impulse-free and stable.

In order to solve Problem RFT, we present the following results, which will be used in the proof of our main result.

Lemma 1[5]： Given the descriptor system

$$\begin{cases} E\dot{x} = Ax + Bu \\ y = Cx \end{cases},$$

then the system is admissible and $\| G \|_\infty < \gamma$ if and only if there exists $X \in \mathbf{R}^{n\times n}$ such that

$$E^T X = X^T E \geq 0 \,,\, A^T X + X^T A + C^T C + \frac{1}{\gamma^2} X^T BB^T X < 0 \,,$$

where $G = C(sE - A)^{-1} B$ and $\gamma > 0$ is a constant.

Lemma 2[6] Given matrices *X, Y, F* of appropriate dimensions, if $F^T F \leq I$, then for arbitrary constant $\lambda > 0$, the following inequality holds：

$$XFY + Y^T F^T X^T \leq \lambda XX^T + \frac{1}{\lambda} Y^T Y \,.$$

Based on Lemmas 1 and 2, we can give a sufficient condition for the existence of solutions to Problem RFT.

Theorem 1：Given the descriptor system (1) with the uncertain parameter matrix satisfying (2), the system (8) with sensor failures and a constant a .

1) Then Problem RFT has a solution if there exists a matrix $P \in \mathbf{R}^{n\times n}$ and an inverse matrix $Q \in \mathbf{R}^{n\times n}$ satisfying the following conditions:

$$E^T P = P^T E \geq 0 \,, \tag{12}$$

$$A^T P + P^T A - P^T (B_F B_F^T - B_{\bar{F}} B_{\bar{F}}^T) P + a^2 C_{\bar{\Lambda}}^T C_{\bar{\Lambda}} + \frac{1}{a^2} P^T (GG^T + a^2 MM^T) P + H^T H + \rho^2 N^T N < 0 \,, \tag{13}$$

$$E^T Q = Q^T E \geq 0 \,, \tag{14}$$

$$\tilde{A}^T Q + Q^T \tilde{A} + Q^T (GG^T + a^2 MM^T + 2a^2 B_{\bar{F}} B_{\bar{F}}^T) Q + \frac{1}{a^2} P^T BB^T P^T - C^T C < 0 \,, \tag{15}$$

where

$$\widetilde{A} = A + \frac{1}{a^2} GG^T P + MM^T P \,. \tag{16}$$

2) When the conditions (14)-(17) are met, one such dynamical compensator of the form of (3) is given by

$$\hat{A} = A + (\frac{1}{a^2} GG^T + M^T M - BB^T) P - Q^{-T} C^T C \,, \tag{17}$$

$$\hat{B} = Q^{-T} C^T \tag{18}$$

and

$$K = -B^T P \,. \tag{19}$$

4. A numerical example

Consider a descriptor system in the form of (1) with the following parameters

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A=\begin{bmatrix}-1&1&0\\1&0&0\\0&0&1\end{bmatrix},\ B=\begin{bmatrix}2&0\\2&1\\1&0\end{bmatrix},\ G=\begin{bmatrix}0.01\\0\\0.1\end{bmatrix},\ C=\begin{bmatrix}6&5&0\\2&7&1\end{bmatrix},\ H=\begin{bmatrix}0&1&0\end{bmatrix},$$

$$\Delta A(\sigma)=ML(\sigma)N=\begin{bmatrix}0.1&0\\0&0.05\\0&0.1\end{bmatrix}\begin{bmatrix}\sin\sigma&0\\0&\cos\sigma\end{bmatrix}\begin{bmatrix}0.1&0.05&0.05\\0&0&0.1\end{bmatrix}.$$

It is easy to verify that $L(\sigma)^T L(\sigma)\le I$. When the second actuator and the first sensor of the system fails, denoted by $F=\{2\}$ and $\Lambda=\{1\}$, then the matrices B_F and C_Λ in form of (6) are

$$B_F=\begin{bmatrix}2&0\\2&0\\1&0\end{bmatrix},\ C_\Lambda=\begin{bmatrix}0&0&0\\2&7&1\end{bmatrix}.$$

We choose

$$P=\begin{bmatrix}1&0&0\\0&2&0\\0&0&1/4\end{bmatrix},\ Q=\begin{bmatrix}17&-4&0\\-4&1&0\\0&0&16\end{bmatrix}.$$

It is easy to prove that the matrices P, Q satisfy (14)-(17) ($a=1/2$). By Theorem 1, we can obtain matrices K, $\hat{B}$ and $\hat{A}$:

$$\hat{A}=10^3\times\begin{bmatrix}-0.2210&-0.3470&-0.0305\\-0.9110&-1.4440&-0.1275\\-0.0021&-0.0044&-0.0013\end{bmatrix},\ \hat{B}=\begin{bmatrix}26&30\\109&127\\0&0.0625\end{bmatrix},K=\begin{bmatrix}-2&-4&-0.25\\0&-2&0\end{bmatrix}.$$

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